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(Residential Autonomous College affiliated to University of Calcutta)

B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2019

SECOND YEAR (BATCH 2017-20)

Date : 28.05.2019 **MATHEMATICS FOR ECONOMICS (General)**

Time : 11.00 am-2.00 pm Paper : IV Full Marks : 75

[Use a separate Answer Book for each group]

Group - A

Answer any five questions from Question No. 1 to 8 : [5×5=25]

1. Is the matrix $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ diagonalizable? Justify. [5]

2. Show that the matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ has no real eigenvalue for $0 < \theta < \frac{\pi}{2}$. [5]

3. Find the eigenvalues of the linear operator $T : P_3(\mathbb{R}) \to P_3(\mathbb{R})$ defined by $T(f(x)) = f'(x) + f(x)$, where $P_3(\mathbb{R})$ is the vector space of all polynomials having degree less than 3. [5]

4. a) Show that if λ be an eigenvalue of a real orthogonal matrix A, then $\frac{1}{\lambda}$ is also an eigenvalue of A.

   b) If the characteristic polynomial of a matrix A is $f(x) = (1-x)^2(1+x)(x+7)$, then find trace of A. [4+1]

5. Is the set of vectors {(1,1,0), (1,-1,1), (-1,1,2)} an orthogonal basis of $\mathbb{R}^3$? Explain. [5]

6. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, show that for every integer $n \geq 3$, $A^n = A^{n-2} + A^2 - I$, where I is the identity matrix of order 3. Hence, determine A^{50}. [5]

7. a) Define convex hull.

   b) Prove that in E^2, the set $X = \{(x, y) : x^2 + y^2 \leq 4\}$ is a convex set. [1+4]

8. Check whether diagonalizable or not- $A = \begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$ [5]

Group - B

Answer any six questions from Question No. 9 to 17 : [6×5=30]

9. Consider the L.P.P. given by

   $3x_1 - 2x_2 + x_3 \leq 2$

   $2x_1 - x_3 \leq -1$

   $x_1 + x_2 - 2x_3 \leq -4,$

   $x_1, x_2, x_3 \geq 0$

   a) Write down the dual of this L.P.P.

   b) Verify that dual of a dual is a primal in this case. [2+3]

10. Solve the following L.P.P. by simplex method.

Maximize, $z = x_1 + 2x_2$

Subject to $3x_1 - x_2 \le 6$

$x_1 + 2x_2 \le 5$

$x_1, x_2 \ge 0$ [5]

11. Solve the following L.P.P.(by any method).

Maximize, $z = 70x_1 + 50x_2 + 35x_3$

Subject to $4x_1 + 3x_2 + x_3 \le 240$

$2x_1 + x_2 + x_3 \le 100$

$-4x_1 + x_2 \le 0$

$x_1, x_2, x_3 \ge 0$

[5]

12. a) Define the basic solution of a system of m linear simultaneous equations in n (n>m) unknowns.

b) Show that the feasible solution $x_1 = 1, x_2 = 1, x_3 = 0$ and $x_4 = 2$ to the system

$x_1 + x_2 + x_3 = 2$

$x_1 + x_2 - 3x_3 = 2$

$2x_1 + 4x_2 + 3x_3 - x_4 = 4$, $x_1, x_2, x_3 x_4 \ge 0$

is not basic. [2+3]

13. a) Define convex set.

b) If x_1, x_2 be real, show that the set given by $X = \{(x_1, x_2) : 9x_1^2 + 4x_2^2 \le 36\}$ is a convex set. [2+3]

14. a) Define slack and surplus variables.

b) Reduce the following minimization problem to a maximization problem in its standard form

Minimize $z = 3x_1 - 2x_2 + 4x_3$

Subject to $x_1 - x_2 + 3x_3 \ge 1$

$2x_1 + 3x_2 - 5x_3 \ge -3$

$4x_1 + 2x_2 \ge 2$

$x_1, x_2, x_3, x_4 \ge 0$. [1+1+3]

15. In a Hospital, meals are served twice to patients. Each 25 grams of the first meal contains 250 units of protein and 500 units of vitamins where as each 25 grams of second meal contains 375 units of protein and 625 units of vitamins. In a days meal each patient should get at least 6000 units of protein and 11000 units of vitamins. Cost of each 100 grams of first meal is Rs. 12.00 and that of second is Rs. 15.00. Formulate an L.P.P. to minimize the cost of food for each day. [5]

16. Use two phase simplex method to solve the following L.P.P.

Maximize $z = 5x_1 + 3x_2$

Subject to $3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$, $x_1, x_2 \ge 0$. [5]

17. Use duality to find the optimal solution, if any, of the following L.P.P.

Maximize $z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \le 5$

$x_1 + x_2 \le 3$, $x_1, x_2 \ge 0$. [5]

Group – C

Answer any two questions from Question No. 18 to 20 : [2×5=10]

18. Consider the following game.

		Player 2 L	M	R
Player 1	U	1, 2	3, 5	2, 1
	N	0, 4	2, 1	3, 0
	D	-1, 1	4, 3	0, 2

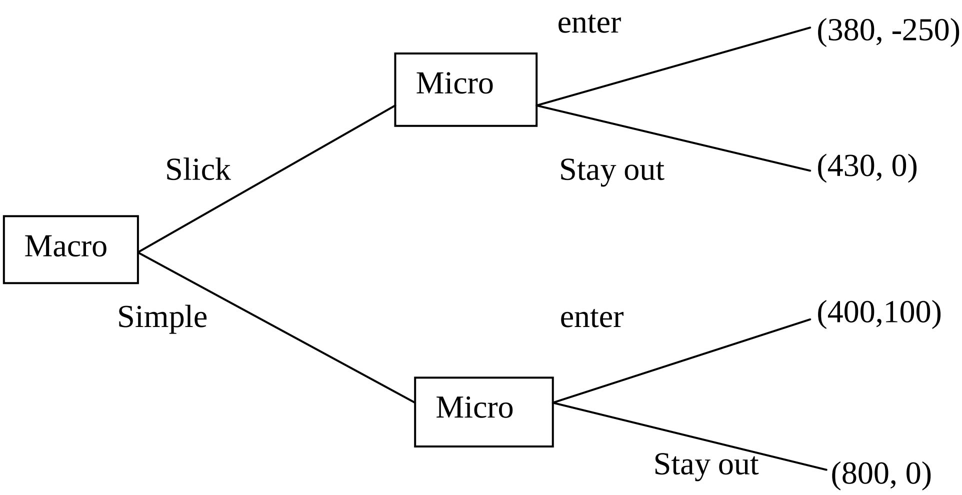
Find a Nash equilibrium of the above game. [5]

19. Solve the game with the following pay-off matrix

$$\text{Player1}\ \begin{array}{c c} & \begin{array}{cc} & \text{Player2} \\ T_1 & T_2 \end{array} \\ \begin{array}{c} S_1 \\ S_2 \end{array} & \begin{pmatrix} -1 & -3 \\ 2 & 2 \end{pmatrix} \end{array}$$

, i.e. find the value and an optimal (mixed) strategy for both the players. [5]

20. Find out the subgame perfect equilibrium for the game :

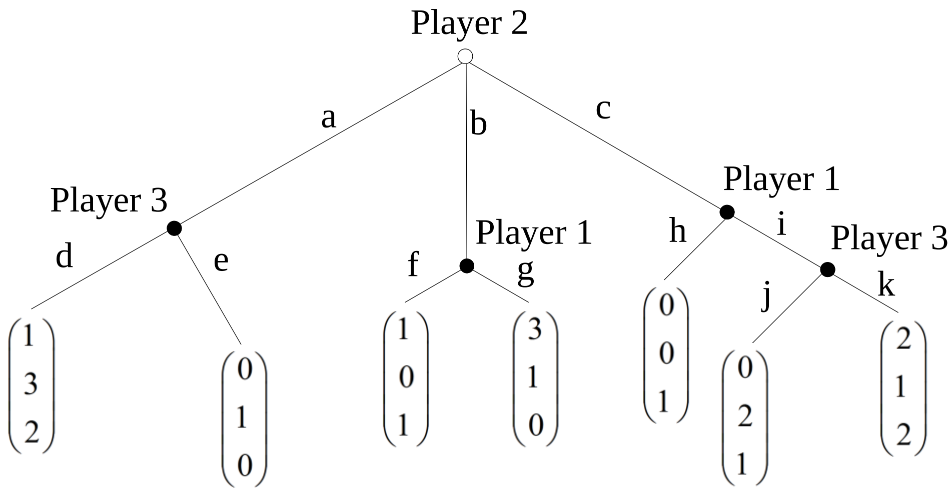


[5]

Answer any one question from Question No. 21 to 22 : [1×10=10]

21. Two firms 1 and 2 produce exactly identical products. Firm 1 produces q_1 units of product and firm 2 produces q_2 units of product. We assume that the market price of the product is - $p(q) = 100 - 2\sqrt{q}$, where $q = q_1 + q_2$. The production cost of the two firms are $c_1(q_1) = q_1 + 10$ and $c_2(q_2) = 2q_2 + 5$. Determine –
    (i) The profit functions of the firms.
    (ii) The Nash equilibrium of the game.
    (iii) The profits of firms at Nash equilibrium. [3+4+3]

22. a) Find the proper subgames of the following game –



b) Distinguish between —

(i) Open and closed Auction

(ii) Sealed bid and oral auction

(iii) English and Dutch Auction [4+(3×2)]

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